Scientific Computing I

Module 2: Population Modelling – Discrete Models

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Winter 2005/2006

Part I: Fibonacci's Rabbits

1. A classical population model

2. Looking for an improved model

Part II: Population Models

3. Discrete vs. Continuous
4. Deterministic vs. Stochastic
5. Spatial and Temporal Resolution
6. Single- vs. Multi-Population
7. Level of Detail
8. Finally: What's the Task?

Part I

Fibonacci's Rabbits

Fibonacci's Rabbits

A pair of rabbits are put in a field.
If rabbits take a month to become mature and then produce a new pair every month, how many pairs will there be in twelve months time?

Leonardo Pisano ("Fibonacci"), A.D. 1202

Model Assumptions

Which assumptions or simplifications have been made?

- we consider pairs of rabbits
- rabbits reproduce exactly once a month
- female rabbits always give birth to a pair of rabbits
- newborn rabbits require one month to become mature
- rabbits don't die
- . . . ?

The Fibonacci Numbers

How many pairs of rabbits are there?

- we start with a newborn pair of rabbits
- after one month: still 1 pair of rabbits (now mature)
- after two months: 2 pairs of rabbits (one mature)
- after three months: 3 pairs of rabbits (two mature)
- after four months: 5 pairs of rabbits (three mature)
- after n months:

$$f_n = f_{n-1} + f_{n-2}, \qquad f_0 = f_1 = 1$$

The Fibonacci Numbers (2)

Now: how many pairs of rabbits **are** there?

- $f_{10} = 55$, $f_{12} = 144$, $f_{18} = 2584$, ...
- exponential growth of rabbits:

$$f_n = \frac{1}{\sqrt{5}} \left(\phi^n - (1 - \phi)^n \right),$$

where $\phi = \frac{1}{2}\left(1 + \sqrt{5}\right) \approx 1.61\ldots$ is the golden section number.

- questions:
  - how accurate is the model?
  - what are its shortcomings?

Wanted: An improved model

Group Work:

Develop an improved model for the growth of a rabbit population!

1. Model assumptions:
   - what assumptions do you want to keep
   - what assumptions do you want to drop or modify
2. Describe your model
3. Describe how to run the simulation
   - starting conditions
   - evolution of the population
   - ...

Comparison of models

Discussion:

What are the differences between the proposed models?

Consider:

- the modelling of the rabbits
- the interaction between rabbits
- the environment (time and space)
- possible external influences

Part II

Classification of Models

Discrete vs. Continuous Models

Discrete Population Modeling:

- count individual rabbits (pairs of rabbits)
- "clocked" evolution of the population: changes occur at discrete points in time or within time intervalls

Continuous Population Modeling:

- population size $\in \mathbb{R}$
- continuous growth or decay

$\Rightarrow$ population size is a function:
$p : \mathbb{R} \to \mathbb{R}, p(x) = \ldots$

Deterministic vs. Stochastic Models

Deterministic Population Modeling:

- fixed birth rate, fixed gender distribution
- model leads to uniform simulation results

Stochastic Population Modeling:

- probability distribution for birth rate and gender
- simulations may lead to different results; both, expected value and aberrations, may be of interest

Spatial and Temporal Resolution

Spatial resolution, only:

- population does not grow or decay
- expanding and spreading of interest

Temporal resolution, only:

- growth and/or decay are of interest
- uniform population distribution in a fixed region

Temporal and spatial resolution

- how does growth/decay affect population distribution?

Single- vs. Multi-Population Models

Single population model:

- population of rabbits
- no other species, but distinction between male/female, healthy/ill, hungry/well-fed, . . . ?

Multi-population:

Example: rabbit population

- competitors: everything that eats carrots!?
- predators: fox, man, . . .
- prey: carrots

$\Rightarrow$ Systems of interacting populations

Level of Detail

Rabbit modelling:

- "pair of rabbits" (mature/non-mature) vs.
- male/female, *x* years old, healthy/ill, hungry/well-fed, . . .

Spatial resolution:

- habitat: friendly/hostile environment
- location of food, competitors, predators, . . .

What Quantities have an Effect?

- what other species have to be included?
- how detailed do we need to model the environment?

Finally: What's the Task?

- find a solution (find all solutions)
- find the best solution (optimization problem)
- analyse solutions:
  Is it unique? How does it depend on input data?
- validate the model:
  quantitatively vs. qualitatively correct?